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ON THE QUESTION OF MEASURING THE VERTICAL TEMPERATURE DISTRIBUTION OF THE ATMOSPHERE FROM SATELLITES

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ABSTRACT

The temperature of the atmosphere cannot be deduced from satellite radiance measurements *alone*; additional information is required. The additional required information is related to the detailed vertical temperature structure of the atmosphere, which is not supplied by the satellite radiance measurements. To deduce the mean temperature (or corresponding mean radiance) in several layers of the atmosphere, the problem may be divided into two parts. One part depends only on the measured satellite radiances; the second part, which depends on the unknown detailed vertical temperature distribution, may be considered as a correction term. Calculations show the magnitude of the two parts. The two parts seem to be related so that mean temperatures (or radiances) can be computed from satellite data with tolerable accuracy with aid of empirical relationships. Other authors have used various assumptions and statistical relations to cope with the required additional temperature information.

1. INTRODUCTION

Methods for deriving the detailed atmospheric vertical temperature structure from a relatively small number of satellite observations of CO_2 emission have been discussed by the authors listed in the "References." These solutions always depend upon information or assumptions in addition to the satellite radiance observations.

For many purposes, it is sufficient to obtain average temperatures in atmospheric layers instead of detailed vertical soundings. For example, to derive the height of standard pressure levels, radiosonde temperature detail is routinely suppressed in favor of average temperatures over fairly deep layers of atmosphere. It is, therefore, desirable to discuss the calculations of average temperatures in layers directly from radiances that might be measured from satellites.

One might perhaps suppose that from N radiation measurements from a satellite the mean temperature in N layers of the atmosphere can be uniquely determined. However, it will be shown below that a unique determination of even mean temperatures of atmospheric layers is not possible; additional assumptions are needed in this case also. This is due to the fact that the calculated mean temperature of atmospheric layers is not only a function of the measured radiances but also of the actual vertical variation of temperature. In general, the variation of temperature with height in any layer is not precisely known; and this produces some uncertainty in the mean temperature calculated from satellite measurements.

This paper discusses the basic reason why the solution for the mean temperature of layers depends on the variation of temperature with height within the layer. Numerical values will be presented to indicate the magnitude of the errors that might result when the vertical variation of temperature is neglected. The possibility of correcting for these errors is also discussed.

The determination of the mean temperature in several layers of the stratosphere and upper troposphere will be discussed. By confining the solution to those regions of the atmosphere, many problems introduced by clouds and water vapor are avoided.

2. CALCULATION OF MEAN TEMPERATURE OF LAYERS

Consider those spectral frequencies in which the atmosphere completely obscures the ground from the satellite.

The radiance, I_i, measured from a satellite at frequency, (ν_i), is then given by Wark and Fleming (1966) as

$$I_i=\int_0^1 B(\nu_i, T)d\tau(\nu_i)=\int_{\ln p_0}^{\ln 0.1} B(\nu_i, T)\,\frac{\partial_\tau(\nu_i, p)}{\partial \ln p}\,d\ln p \qquad (1)$$

where $B(\nu_i, T)$ is the Planck radiance at frequency, ν_i, and temperature, T. The transmittance, $\tau(\nu_i, p)$ is the fraction of the energy, upwelling at the pressure level p, which is transmitted to the satellite through the atmosphere lying above the level p. The "top of the atmosphere" is assumed to be at $p=0.1$ mb; this is an adequate "top" for the 15μ CO_2 band with resolution of about 5 cm^{-1}, since the transmittance is very nearly unity for all spectral regions at $p=0.1$ mb.

For purposes of illustration, suppose measurements are made at two frequencies, a and b. Assume that the Planck functions, B, have been normalized (Wark and Fleming, 1966) to a given reference frequency, ν_r, and that we require the average B, for two layers of the atmosphere. (The average Planck function, $\overline{B}$, will be closely related to the average temperatures in the layers.)

Therefore, from equation (1),

$$I_a=\left[\overline{B\,\frac{\partial\tau_a}{\partial \ln p}}\,\Delta \ln p\right]_H+\left[\overline{B\,\frac{\partial\tau_a}{\partial \ln p}}\,\Delta \ln p\right]_L \qquad (2)$$

and

$$I_b=\left[\overline{B\,\frac{\partial\tau_b}{\partial \ln p}}\,\Delta \ln p\right]_H+\left[\overline{B\,\frac{\partial\tau_b}{\partial \ln p}}\,\Delta \ln p\right]_L \qquad (3)$$

where H refers to the higher layer and L refers to the lower layer. The normalization factors for frequency have been incorporated into I_a and I_b. The averages have been performed with respect to $\ln p$. Let

$$B=\overline{B}+B', \text{ and } \frac{\partial \tau}{\partial \ln p}=\overline{\left(\frac{\partial \tau}{\partial \ln p}\right)}+\left(\frac{\partial \tau}{\partial \ln p}\right)' \tag{4}$$

where the primes denote deviations of a quantity at a particular level from its average value in a layer.

Then,

$$\overline{B\left(\frac{\partial \tau_a}{\partial \ln p}\right)}=\overline{B}\,\overline{\left(\frac{\partial \tau_a}{\partial \ln p}\right)}+\overline{B'\left(\frac{\partial \tau_a}{\partial \ln p}\right)'}. \tag{5}$$

This follows from the substitution of equation (4) into the left side of equation (5), since

$$\overline{B'}=0 \text{ and } \overline{\left(\frac{\partial \tau_a}{\partial \ln p}\right)'}=0.$$

Since

$$\overline{\left(\frac{\partial \tau_a}{\partial \ln p}\right)}\Delta \ln p=\Delta \tau_a,$$

equation (2) becomes

$$I_a=[\overline{B}\Delta\tau_a]_H+[\overline{B}\Delta\tau_a]_L+\left[\overline{B'\left(\frac{\partial \tau_a}{\partial \ln p}\right)'}\Delta \ln p\right]_H + \left[\overline{B'\left(\frac{\partial \tau_a}{\partial \ln p}\right)'}\Delta \ln p\right]_L \tag{6}$$

with a corresponding equation from equation (3) for frequency b. Here, $(\Delta\tau_a)_H=1-\tau_D$; $(\Delta\tau_a)_L=\tau_D-0=\tau_D$; τ_D is the value of transmittance at the boundary that divides the upper and lower layers.

The solutions for $\overline{B}_H$ and $\overline{B}_L$ are, in matrix notation,

$$\begin{pmatrix}\overline{B}_H\\ \overline{B}_L\end{pmatrix}=\mathbf{A}^{-1}\begin{pmatrix}I_a\\ I_b\end{pmatrix}-\mathbf{A}^{-1}\begin{pmatrix}V_a\\ V_b\end{pmatrix}. \tag{7}$$

Here, $\mathbf{A}^{-1}$ is the inverse of matrix $\mathbf{A}$;

$$\mathbf{A}=\begin{pmatrix}\Delta\tau_{aH} & \Delta\tau_{aL}\\ \Delta\tau_{bH} & \Delta\tau_{bL}\end{pmatrix}$$

and V_a is the sum of the last two terms in equation (6); V_b is the corresponding term in the equation involving frequency b.

Equation (7) shows that the B's depend on a linear combination of the I's. To that extent the solution is similar to other proposed solutions (Wark and Fleming, 1966; Westwater and Strand, 1968; Rodgers, 1966; Chahine, 1968; and Smith, 1969), although the coefficients of the I's will be different. In a sense, the second term in equation (7) can be considered a correction term, after the first term, involving the radiances, is used to find the $\overline{B}$'s.

From equation (7), it is evident that even without errors in the measured radiances, and even if only two average temperatures are sought from two spectral measurements, additional information is required in order to evaluate $\overline{B}_H$ or $\overline{B}_L$. The additional information required, in addition to the radiances, is a knowledge of the magnitude of the terms V_a and V_b. These involve the covariance between B' and $(\partial\tau/\partial\ln p)'$, at the two frequencies. Since the vertical variation of the transmittances is assumed known, the additional information requires an assumption about the vertical variation of B or of T. This covariance term may perhaps be estimated from climatological data, or from synoptic data where radiosonde measurements exist. Perhaps it is sufficiently well correlated with $\overline{B}$ itself or with the radiances. At any rate, some estimate of it is required.

The reasoning that leads to equation (7) can be extended to any number of measurements. If the number of measurements could become very large and extend with sufficient weight to the 0.1-mb level, the sum of terms involving $\overline{B'\left(\frac{\partial \tau}{\partial \ln p}\right)'}$ would approach zero. However, because of practical instrument limitations, the number of independent radiance measurements in the CO_2 bands is relatively small in instruments now being designed. For example in the Satellite Infrared Spectrometer (SIRS) now installed on Nimbus III, there are seven "channels" in the 15μ CO_2 band plus one channel in the 11μ atmospheric "window." Of these, about four channels can be used to sense the atmosphere down to about 300 mb (fig. 3 in Wark and Fleming, 1966); that is, the radiances in only four channels are influenced directly by the atmosphere above about 300 mb and almost not at all by the atmosphere below 300 mb.

Thus if we confine our study to the determination of mean temperature in four layers of the atmosphere lying between 0.1 mb and about 300 mb, we can extend the reasoning that led to equation (7) to obtain

$$\overline{\mathbf{B}}=\mathbf{A}^{-1}(\mathbf{I})-\mathbf{A}^{-1}(\mathbf{V}). \tag{8}$$

Now, $\overline{\mathbf{B}}$ is a vector with four values; i.e., one mean value for each of four atmospheric layers. There are four radiances, I; the matrix $\mathbf{A}$ is a 4×4 matrix. Finally V, for each frequency, is a sum of four terms of the form $\overline{B'\left(\frac{\partial \tau_a}{\partial \ln p}\right)'}$, one arising from each of the four layers.

For ease in discussing equation (8), define

$$\overline{B}N=BN1-BN2$$

where

$$BN1=\sum_{m=1}^{4} a_{mN}I_m$$

and

$$BN2=\sum_{m=1}^{4} a_{mN}V_m.$$

TABLE 1.—*Values of terms $\overline{B}N$, $BN1$, and $BN2$ (ergs/cm²/sec/ster/cm⁻¹)*

p(mb)	Key West	Little Rock	Seattle	Churchill
0.1				
$BH1$	71.5	69.6	54.1	45.7
$BH2$	−4.5	−5.5	−9.3	−13.5
$\overline{B}H$	76.0	75.1	63.4	59.2
18.8				
$BI1$	31.8	32.9	34.0	21.7
$BI2$	−5.2	−5.8	−4.7	−6.7
$\overline{B}I$	37.0	38.7	38.7	28.4
78.4				
$BM1$	25.2	36.2	38.0	37.4
$BM2$	−4.0	−0.1	+0.0	+3.7
$\overline{B}M$	29.2	36.2	38.0	33.7
145.5				
$BL1$	49.4	42.7	40.4	37.6
$BL2$	+1.9	+0.5	+0.2	−0.5
$\overline{B}L$	47.5	42.2	40.2	38.1
327				

The four layers are given by N which takes on designators of H (high), I (intermediate), M (middle), L (lowest) layers.

For example $\overline{B}H=BH1-BH2$, etc.

3. NUMERICAL RESULTS

Unfortunately, four layers are not sufficient to reduce the correction term, involving V, to zero. This was deduced from a numerical analysis of the four soundings considered by Wark and Fleming (1966, figs. 5–8), namely Key West, Little Rock, Seattle, and Churchill. For this numerical study computed radiances, emerging from the atmosphere above 300 mb, were used at the following frequencies: 669, 677.5, 691, and 699 cm⁻¹. For the present study, computed radiances are preferable to measured ones, since instrumental error can be eliminated as a source of error in the calculations.

The atmosphere between 0.1 mb and about 300 mb can be divided into four layers in many different ways. Numerical experiments were performed to reduce the magnitudes of the correction terms in the two lowest layers to small values. An experiment that gave a good result divided the atmosphere at the following pressure levels: 0.1, 18.8, 78.4, 145.5, and 326.9 mb. (These strange pressures appear when the atmosphere is divided into 200 layers of equal logarithm of pressure.)

The values of $BN1$, $BN2$, and $\overline{B}N$ are shown in table 1. The table shows that $BN2$, the second term in equation (8), is a substantial part of $\overline{B}N$, especially in the upper layers. In this case, a significant part of $BN2$ was contributed by the uppermost layer, so that the reduction of the term $\mathbf{A}^{-1}(\mathbf{V})$ might perhaps require the addition of frequencies most sensitive in the region between 0.1 mb and 80 mb.

If $BN2$ can be considered a correction term to $BN1$ in order to attain $\overline{B}N$, it might be useful to approximate

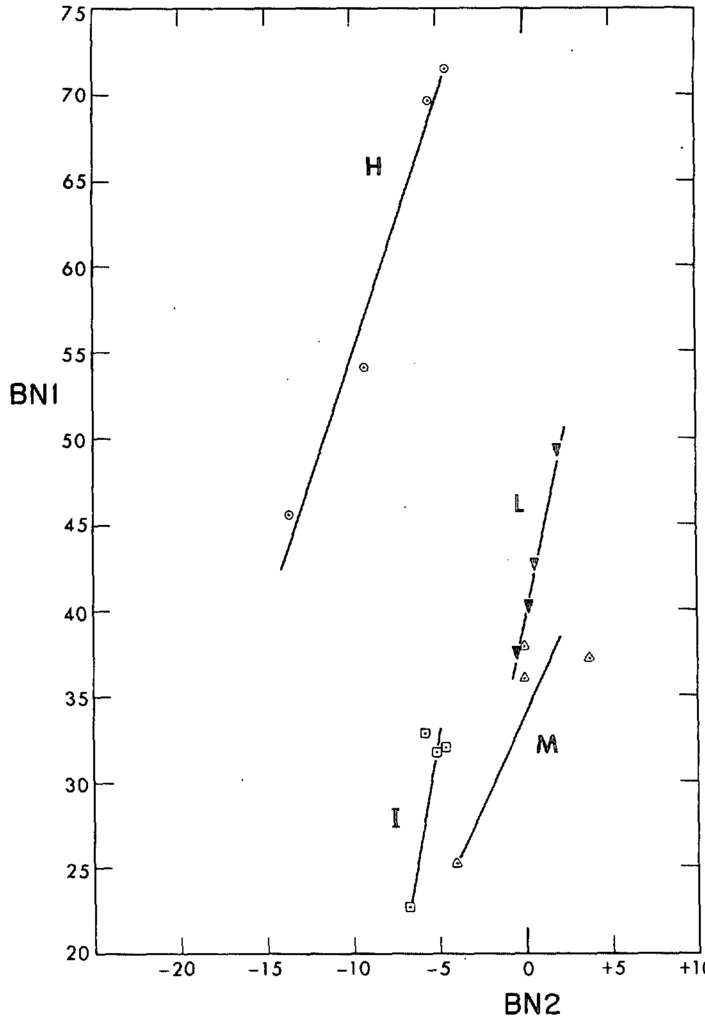

FIGURE 1.—The relationship between $BN1$ and $BN2$, see equation (8) and table 1. Line labels mean the following: H=highest layer, I=next highest (or intermediate), M (middle), and L=lowest. $BN1$ is the part of the computed average layer temperature that is directly related to the satellite radiances and the CO_2 transmittances. $BN2$ is the part of the computed average layer temperature that depends on the detailed atmospheric temperature and the detailed CO_2 transmittances. The units of $BN2$ and $BN1$ are ergs/cm²/sec/ster/cm⁻¹.

$BN2$ from $BN1$, which depends directly on the radiances. One might expect that $BN2$ would be related to $BN1$. They both contain the same coefficients, which are the elements of the inverse matrix, $\mathbf{A}^{-1}$. Moreover, the factors, V, which appear in $BN2$, also appear in I in equation (6). To see whether $BN2$ can be approximated from $BN1$, these two variables were plotted in figure 1. We do indeed find that they vary together. The lines, fitted by eye, yield $BH2$, $BI2$, and $BL2$ within one unit of B. The fit for $BM2$ is not as good; the deviation would be ±two units in B. (One unit in B corresponds to about 1°K, when B=35.)

It might also be possible to approximate $BN2$ in the form

$$BN2=\sum_{n=1}^{4} c_{nN} I_n \tag{9}$$

where the c's are determined by regression relations. For it is easy to show that if

$$\overline{B}N=\sum_{n=1}^{4} k_{nN} I_n \tag{10}$$

as the several authors previously cited propose, then, because from equation (8) $BN1$ is also of that form, $BN2$ must be of the form given by equation (9).

4. CONCLUSIONS

The results discussed above indicate that some assumption about the atmospheric structure must be made to evaluate the atmospheric temperature. Many methods (Wark and Fleming, 1966; Westwater and Strand, 1968; Rodgers, 1966; Smith, 1969) utilize the statistics of the atmospheric temperature structure; i.e., they use the fact that the temperature at one level in the atmosphere is highly correlated with temperature at adjacent levels and even with levels far removed. Other methods assume that the atmosphere is made up of straight line or curved segments between temperatures that have been evaluated by iterative or other methods (Chahine, 1968; House and others, 1968; and Wark, 1961).

These studies (except Wark, 1961) generally attempted to find, not the mean temperature in layers, but more detailed structure of the atmosphere. However, even to deduce the mean temperature in layers, an assumption about the atmospheric structure must be made. Wark (1961), for example, assumed that the Planck function, B, was linear with $\log p$ in each layer.

One method which might be attempted treats the second term on the right of equation (8) as a correction term.

This, together with a relation of the type suggested in figure 1, can perhaps be used to estimate $BN2$ from $BN1$. Since $BN1$ is obtained directly from the radiances, $\overline{B}N$ would then also depend only on the radiances.

Still other methods, always involving some assumptions which directly or indirectly cope with the terms involving V, can of course be designed.

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CORRECTION NOTICE

Vol. 96, No. 10, Oct. 1968, p. 736, caption of figure 1(a) and 1(b): "absorption" should be replaced by "slab absorptivity" and add "The absorptivity of a slab is computed from the absorptivity of a column by using the following formula,

$$a_f(u)=a_t(1.66u)$$

where a_f and a_t are the absorptivities of slab and column, respectively, and u is optical thickness at STP; ordinate (a_f), abscissa $(\log u)$"; p.739, legend of figure 8: symbols for plotting (M–S) and $(R–W)_2$ should be interchanged.